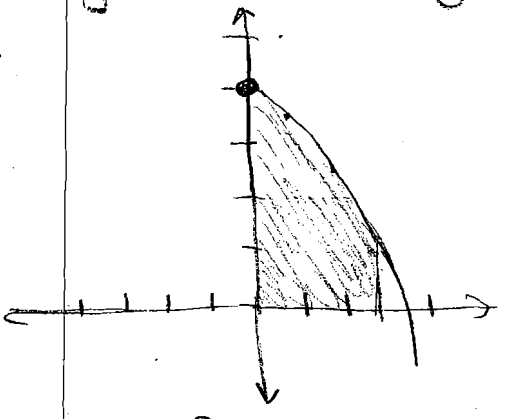
Unit 6 wrksht 1

odds

① $y = 4 - \frac{1}{3}x^2$, $y = 0$, $x = 0$, $x = 3$

$[0, 3]$



$$A = \int_0^3 4 - \frac{1}{3}x^2$$

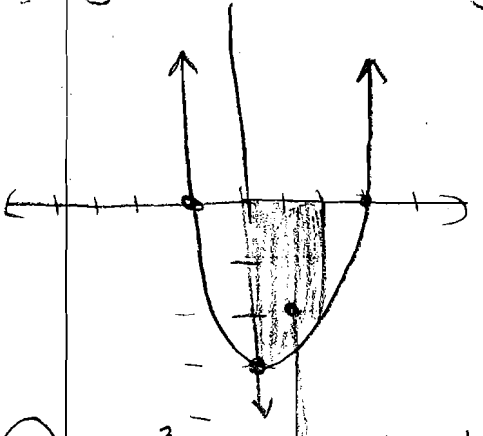
$$\left[4x - \left(\frac{1}{3}\right)\left(\frac{x^3}{3}\right)\right]_0^3$$

$$\left[4(3) - \frac{3^3}{9}\right] - [0 - 0]$$

$$12 - 3$$

$$= 9$$

③ $y = x^2 - 2x - 3$, $y = 0$, $x = 0$, and $x = 2$



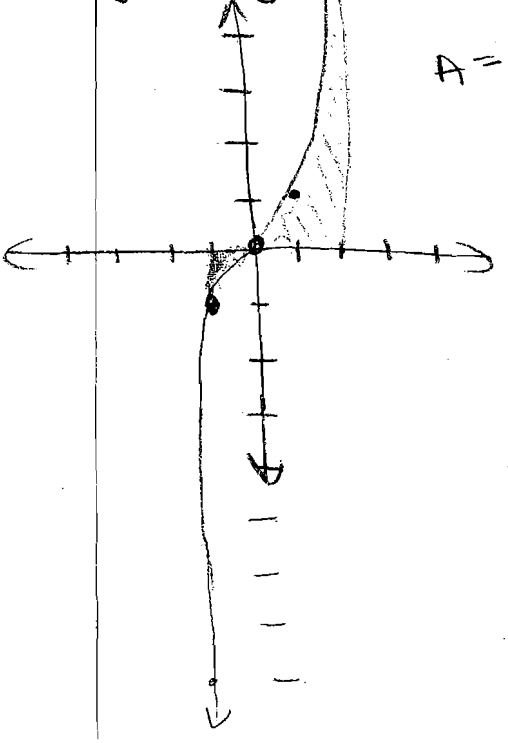
$$A = -\int_0^2 x^2 - 2x - 3$$

$$-\left[\frac{x^3}{3} - \frac{2x^2}{2} - 3x\right]_0^2$$

$$F(2) - F(0)$$

$$-\left[-22/3 - 0\right] = \boxed{22/3} \text{ or } (7\,1/3 \text{ or } 7.\overline{3})$$

⑤ $y = x^3$, $y = 0$, $x = -1$, and $x = 2$



$$A = -\int_{-1}^0 x^3\,dx + \int_0^2 x^3\,dx$$

$$-\left[\frac{x^4}{4}\right]_{-1}^0 + \left[\frac{x^4}{4}\right]_0^2$$

$$-[F(0) - F(-1)] + [F(2) - F(0)]$$

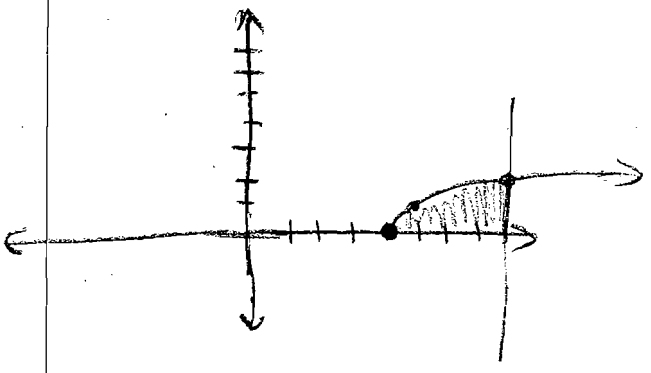
$$-[0 - 1/4] + [\tfrac{16}{4} - 0]$$

$$1/4 + 16/4 = 17/4$$

$$A = 4\,1/4 \text{ or } 4.25$$

Unit 6 worksht 1
odds

(7) $y = \sqrt{x-4}$, $y = 0$, and $x = 8$



$$A = \int_4^8 (x-4)^{1/2}\,dx$$

$u = x-4$ $\quad x=4 \Rightarrow u=0$

$du = 1\,dx$ $\quad x=8 \Rightarrow u=4$

$$\int_0^4 u^{1/2}\,du$$

$$\left[\frac{2}{3}u^{3/2}\right]_0^4$$

$$\frac{16}{3} - 0$$

$A = \frac{16}{3}$ or $5\frac{1}{3}$ or $5.\overline{3}$

(9) $y = x^2$ and $y = x+2$

$x^2 = x+2$

$x^2 - x - 2 = 0$

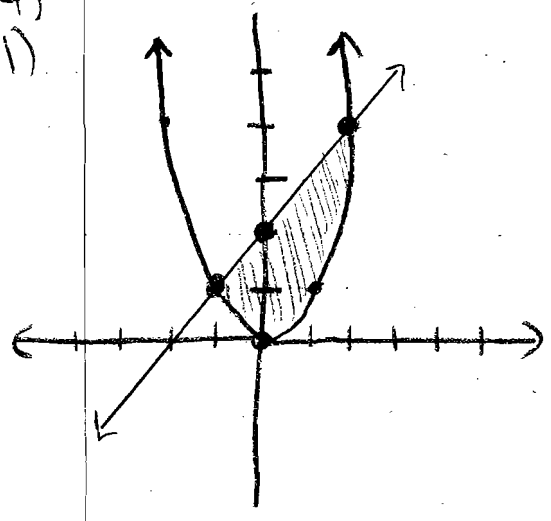
$(x-2)(x+1)$

$x = 2, x = -1$

POI

$(2, 4)$

$(-1, 1)$



$$A = \int_{-1}^{2} [(x+2) - x^2]\,dx$$

$$A = \int_{-1}^{2} (-x^2 + x + 2)\,dx$$

$$A = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$$

$A = F(2) - F(-1)$

$A = 10/3 - (-7/6)$

$A = 9/2$ or $4\frac{1}{2}$ or 4.5

Unit 6 worksht 1
odds

(11) $y = x^2 - 4x$ and $y = -x^2$ (lower, upper)

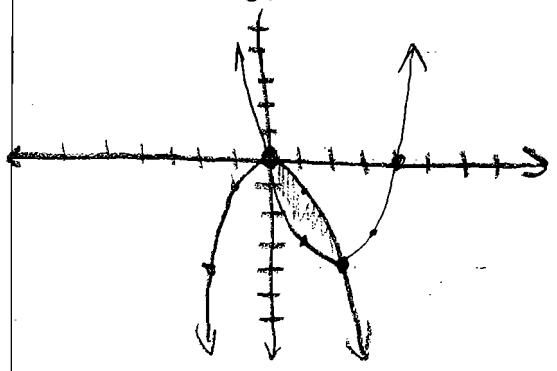
$x^2 - 4x = -x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0 \quad x = 2$

POI
(0, 0)
(2, -4)



$\int_0^2 [-x^2 - (x^2 - 4x)]\,dx$

$\int_0^2 (-2x^2 + 4x)\,dx$

$\left[\frac{-2x^3}{3} + \frac{4x^2}{2}\right]_0^2$

$F(2) - F(0)$

$8/3 - 0$

$A = 8/3$ OR $2\frac{2}{3}$ OR $2.\overline{6}$

(13) $x = 6y - y^2$ and $x = 0$

$y^2 - 6y = -x$

$y^2 - 6y + 9 = -x + 9$

$(y-3)^2 = -x + 9$

$y - 3 = \pm\sqrt{-x+9}$

$y = \pm\sqrt{9-x} + 3$

$[0, 6]$ ← curve $x = 0$ y-axis

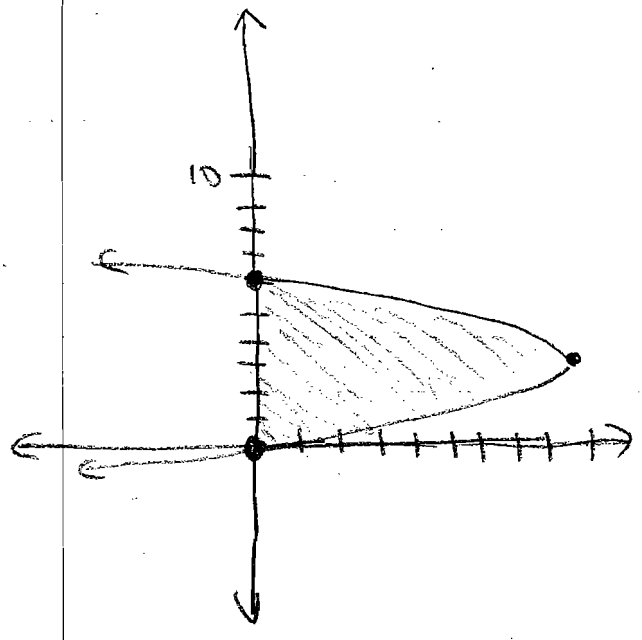
$A = \int_0^6 (6y - y^2)\,dy$

$A = \left[\frac{6y^2}{2} - \frac{y^3}{3}\right]_0^6$

$F(6) - F(0)$

$A = 36 - 0$

$\boxed{A = 36}$



Unit 6 wrksht 1
odds

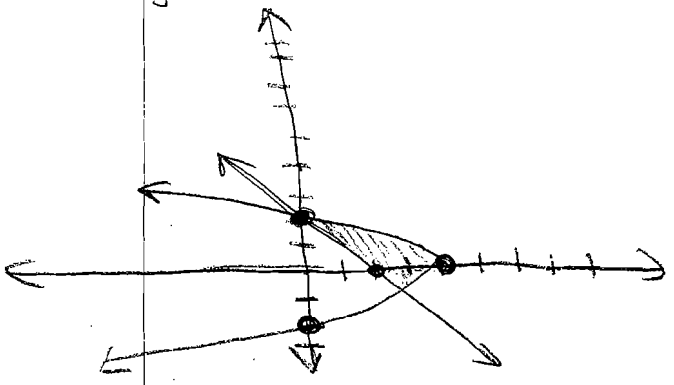
(15) $x = 4 - y^2$ and $x + y - 2 = 0$

$y^2 = 4 - x$

$y = \pm\sqrt{4-x}$

$y = -x + 2$ for graphing

$x = -y + 2$ for antiderivative



$4 - y^2 = -y + 2$

$y^2 - y - 2 = 0$

$(y - 2)(y + 1) = 0$

$y = 2, y = -1$

$A = \int_{-1}^{2} [(4 - y^2) - (-y + 2)]\,dy$

$A = \int_{-1}^{2} (-y^2 + y + 2)\,dy$

$A = [-y^3/3 + y^2/2 + 2y]_{-1}^{2}$

$A = F(2) - F(-1)$

$A = 10/3 - (-7/6) =$ (9 1/2 or 4 1/2 or 4.5) $= A$

(17) $y^2 - 2x = 0$ and $y^2 + 4x - 12 = 0$

$y^2 = 2x$

$y = \pm\sqrt{2x}$

$y^2 = -4x + 12$

$y = \pm\sqrt{12 - 4x}$

$y^2 = 2x$

$2x = -4x + 12$

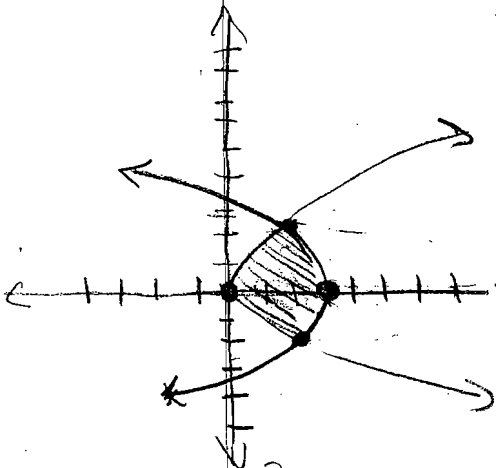
$6x = 12$

$x = 2$

$y^2 = 2x$

$y^2 = 4$

$y = \pm 2$



$A = \int_{-2}^{2} [(-1/4y^2 + 3) - (1/2y^2)]\,dy$

$A = \int_{-2}^{2} (-3/4y^2 + 3)\,dy$

$A = [-3/4(y^3/3) + 3y]_{-2}^{2}$

$[-y^3/4 + 3y]_{-2}^{2}$

$A = F(2) - F(-2)$

$A = 4 - (-4)$

$\boxed{A = 8}$

$y^2 - 2x = 0$

$2x = y^2$

$x = 1/2\,y^2$

$y^2 + 4x - 12 = 0$

$4x = -y^2 + 12$

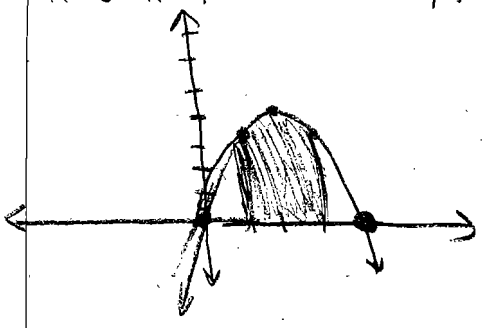
$x = -1/4\,y^2 + 3$

Unit 6 workshT 1
even

② $y = 4x - x^2$, $y = 0$, $x = 1$, and $x = 3$

$x(4-x)$

$x=0 \quad x=4$



$$A = \int_1^3 (4x - x^2)\,dx$$

$$\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_1^3$$

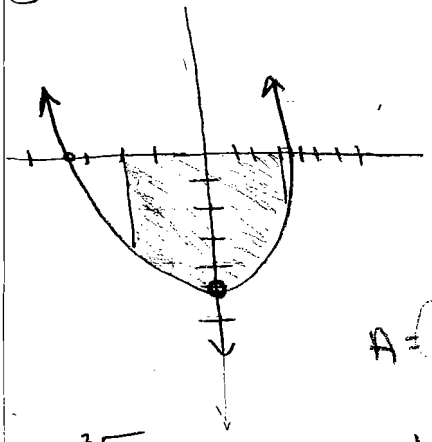
$A = F(3) - F(1)$

$A = 9 - 5\frac{1}{3}$

$A = 3\frac{2}{3}$ or $7\frac{1}{3}$ or $7.\overline{3}$

④ $y = \frac{1}{2}(x^2 - 10)$, $y = 0$, $x = -2$, and $x = 3$

$y = \frac{1}{2}x^2 - 5$



$$A = \int_{-2}^{3} (\tfrac{1}{2}x^2 - 5)\,dx$$

$$A = \left[\frac{\frac{1}{2}x^3}{3} - 5x\right]_{-2}^{3}$$

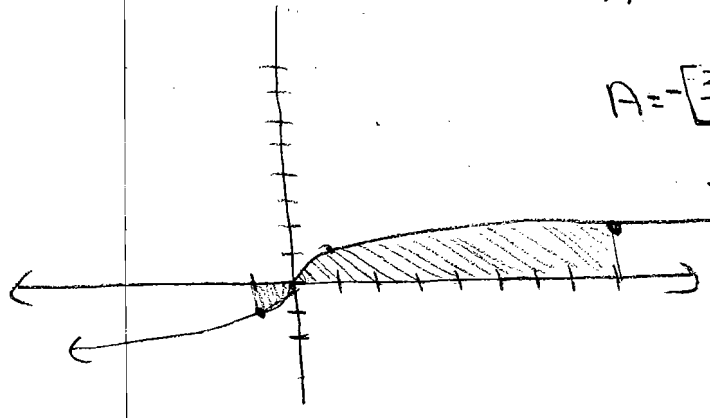
$A = F(3) - F(-2)$

$A = [-10.5 - 8\frac{1}{3}]$

$A = -[-11\frac{5}{6}]$

$A = 11\frac{5}{6}$ or $19\frac{1}{6}$ or 19.1667

⑥ $y = \sqrt[3]{x}$, $y = 0$, $x = -1$, and $x = 8$



$$A = -\int_{-1}^{0} x^{1/3}\,dx + \int_0^8 x^{1/3}\,dx$$

$$A = -\left[\tfrac{3}{4}x^{4/3}\right]_{-1}^{0} + \left[\tfrac{3}{4}x^{4/3}\right]_0^8$$

$$-[F(0) - F(-1)] + [F(8) - F(0)]$$

$A = -(-3/4) + 12$

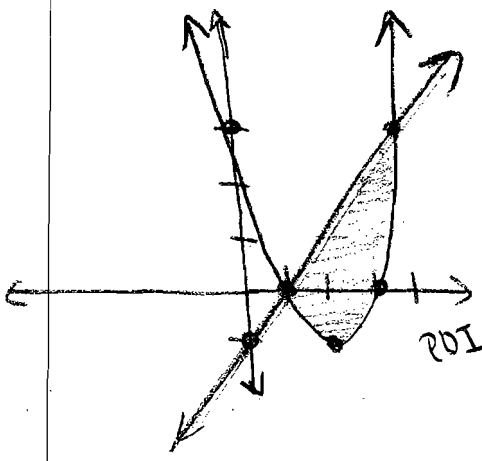
$A = 12\frac{3}{4}$ or 12.75

Unit 6 worksheet 1
even

(8) $y = x^2 - 4x + 3$ and $x - y - 1 = 0$

$y = x - 1$

$A = \int_1^4 [(x-1) - (x^2 - 4x + 3)]\,dx$



$x^2 - 4x + 3 = x - 1$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

POI $x = 4$ $x = 1$

$A = \int_1^4 (-x^2 + 5x - 4)\,dx$

$\left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^4$

$A = F(4) - F(1)$

$A = 8/3 - (-11/6)$

$A = 9/2$ or $4\frac{1}{2}$ or 4.5

(10) $y = 2\sqrt{x}$, $y = 2x - 4$, and $x = 0$

$\frac{y}{2} = \sqrt{x}$

$x = \frac{1}{4}y^2$

$y + 4 = 2x$

$\frac{1}{2}y + 2 = x$

$\frac{1}{4}y^2 = \frac{1}{2}y + 2$

$y^2 = 2y + 8$

$y^2 - 2y - 8 = 0$

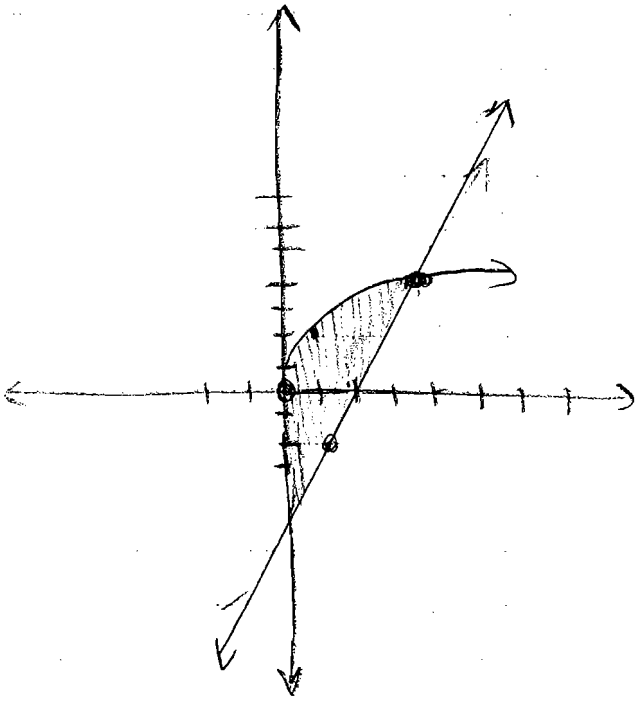
$(y-4)(y+2) = 0$

$y = 4$ $y = -2$

$x = \frac{1}{2}y + 2$

$x = \frac{1}{2}(4) + 2 = 4$

$x = \frac{1}{2}(-2) + 2 = 1$



$A = \int_0^4 [2x^{1/2} - (2x - 4)]\,dx$

$A = \left[\frac{2x^{3/2}}{3/2} - \frac{2x^2}{2} + 4x\right]_0^4$

$A = \left[\frac{4}{3}x^{3/2} - x^2 + 4x\right]_0^4$

$A = F(4) - F(0)$

$A = 32/3 - 0$

$A = \frac{32}{3}$ or $10\frac{2}{3}$ or $10.\overline{6}$

Unit 6 worksht 1
even

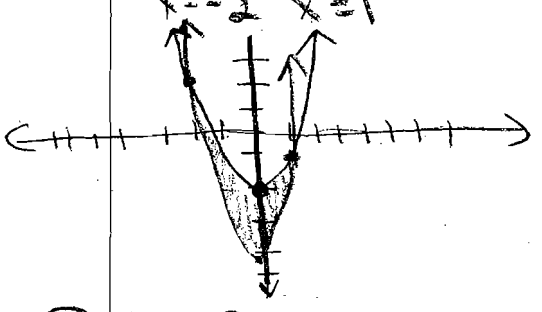
(12) $y = x^2 - 2$ and $y = 2x^2 + x - 4$

$x^2 - 2 = 2x^2 + x - 4$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$



$A = \int_{-2}^{1} [(x^2 - 2) - (2x^2 + x - 4)]\,dx$

$A = \int_{-2}^{1} [-x^2 - x + 2]\,dx$

$A = \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^{1}$

$A = F(1) - F(-2)$

$A = 7/6 - (-10/3)$

$A = 9/2$ or $4\frac{1}{2}$ or 4.5

(14) $x = -y^2 + y + 2$ and $x = 0$

$-y^2 + y + 2 = x$

$y^2 - y = -x + 2$

$y^2 - y + 1/4 = -x + 2 + 1/4$

$(y - 1/2)^2 = -x + 9/4$

$y - 1/2 = \pm\sqrt{-x + 9/4}$

$y = \pm\sqrt{9/4 - x} + 1/2$

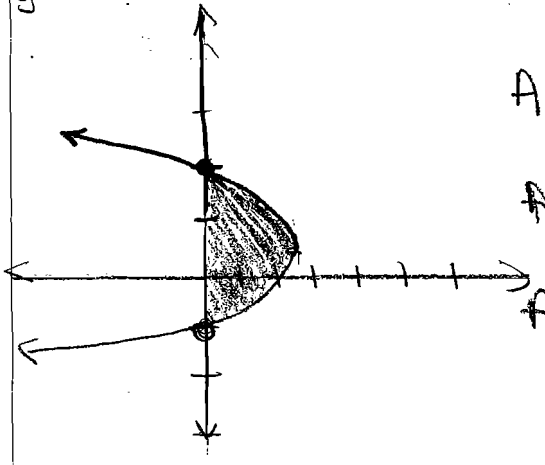
y-intercepts

$-y^2 + y + 2 = 0$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$



$A = \int_{-1}^{2} (-y^2 + y + 2)\,dy$

$A = \left[-\frac{y^3}{3} + \frac{y^2}{2} + 2y\right]_{-1}^{2}$

$A = F(2) - F(-1)$

$A = 10/3 - (-7/6)$

$A = 9/2$ or $4\frac{1}{2}$ or 4.5

Unit 6 wksht 1

even

(16) $x = y^2 - 3y$ and $x - y + 3 = 0$

$y = -x + 3$

or

$x = y - 3$

graph for antiderivative

Intersect

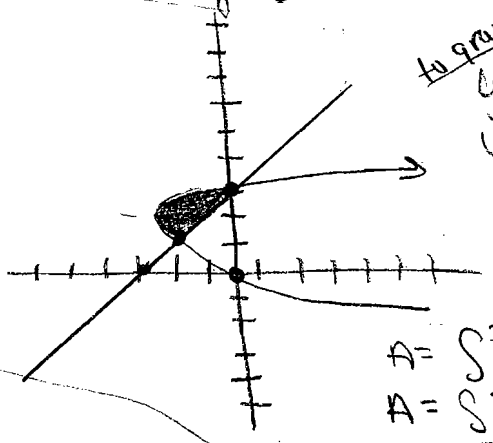
$y^2 - 3y = y - 3$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3, y = 1$

$(0,3)$ and $(-2,1)$



to graph

$y^2 - 3y = x$

$y^2 - 3y + 9/4 = x + 9/4$

$(y - 3/2)^2 = x + 9/4$

$y - 3/2 = \pm\sqrt{x + 9/4} + 3/2$

$A = \int_1^3 [(y-3) - (y^2 - 3y)]\,dy$

$A = \int_1^3 (-y^2 + 4y - 3)\,dy$

$A = \left[-y^3/3 + 4y^2/2 - 3y\right]_1^3$

$A = F(3) - F(1)$

$0 - (-4/3)$

$A = 4/3$ or $1\,1/3$ or $1.\overline{3}$

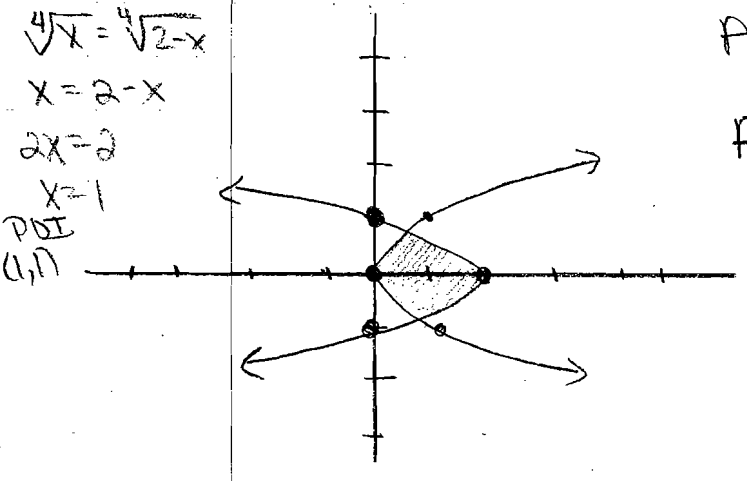
(18) $x = y^4$ and $x = 2 - y^4$

$y = \pm\sqrt[4]{x}$

$y^4 = 2 - x$

$y = \pm\sqrt[4]{2-x}$

for graphing



$\sqrt[4]{x} = \sqrt[4]{2-x}$

$x = 2 - x$

$2x = 2$

$x = 1$

POI $(1,1)$

$A = \int_{-1}^{1} [(2 - y^4) - (y^4)]\,dy$

$A = \int_{-1}^{1} (2 - 2y^4)\,dy$

$A = \left[2y - \frac{2y^5}{5}\right]_{-1}^{1}$

$F(1) - F(-1)$

$A = 8/5 - (-8/5)$

$A = 16/5$ or $3\,1/5$ or 3.2